**Cambridge International Examinations**
Cambridge International Advanced Level

**MATHEMATICS** **9709/33**

Paper 3 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

## Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

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|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no “follow through” from a previous error is allowed) |
| CWO | Correct Working Only – often written by a ‘fortuitous’ answer |
| ISW | Ignore Subsequent Working |
| SOI | Seen or implied |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

**<u>Penalties</u>**

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|---|---|
| MR –1 | A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA –1 | This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting. |

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|---|---|---|---|
| **1** | Use law of the logarithm of a quotient | **M1** | |
| | Remove logarithms and obtain a correct equation, e.g. $e^z = \dfrac{y+2}{y+1}$ | **A1** | |
| | Obtain answer $y = \dfrac{2-e^z}{e^z-1}$, or equivalent | **A1** | [3] |
| **2** | Use correct quotient or product rule | **M1** | |
| | Obtain correct derivative in any form | **A1** | |
| | Use Pythagoras to simplify the derivative to $\dfrac{1}{1+\cos x}$, or equivalent | **A1** | |
| | Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent | **A1** | [4] |
| **3** | Use the tan *2A* formula to obtain an equation in tan $\theta$ only | **M1** | |
| | Obtain a correct horizontal equation | **A1** | |
| | Rearrange equation as a quadratic in tan $\theta$, e.g. $3\tan^2\theta + 2\tan\theta - 1 = 0$ | **A1** | |
| | Solve for $\theta$ (usual requirements for solution of quadratic) | **M1** | |
| | Obtain answer, e.g. 18.4° | **A1** | |
| | Obtain second answer, e.g. 135° , and no others in the given interval | **A1** | [6] |
| **4 (i)** | Commence division by $x^2 - x + 2$ and reach a partial quotient $4x^2 + kx$ | **M1** | |
| | Obtain quotient $4x^2 + 4x + a - 4$ or $4x^2 + 4x + b/2$ | **A1** | |
| | Equate *x* or constant term to zero and solve for *a* or *b* | **M1** | |
| | Obtain $a = 1$ | **A1** | |
| | Obtain $b = -6$ | **A1** | [5] |
| **(ii)** | Show that $x^2 - x + 2 = 0$ has no real roots | **B1** | |
| | Obtain roots $\frac{1}{2}$ and $-\frac{3}{2}$ from $4x^2 + 4x - 3 = 0$ | **B1** | [2] |
| **5 (i)** | State equation $\dfrac{dy}{dx} = \frac{1}{2}xy$ | **B1** | [1] |
| **(ii)** | Separate variables correctly and attempts to integrate one side of equation | **M1** | |
| | Obtain terms of the form $a\ln y$ and $bx^2$ | **A1** | |
| | Use $x = 0$ and $y = 2$ to evaluate a constant, or as limits, in expression containing $a$ ln y or $bx^2$ | **M1** | |
| | Obtain correct solution in any form, e.g. $\ln y = \frac{1}{4}x^2 + \ln 2$ | **A1** | |
| | Obtain correct expression for *y*, e.g. $y = 2e^{\frac{1}{4}x^2}$ | **A1** | [5] |
| **(iii)** | Show correct sketch for $x \geqslant 0$. Needs through (0, 2) and rapidly increasing positive gradient. | **B1** | [1] |

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| **6 (i)** | State or imply $\mathrm{d}u = \dfrac{1}{2\sqrt{x}}\mathrm{d}x$ | **B1** | |
| | Substitute for $x$ and d$x$ throughout | **M1** | |
| | Justify the change in limits and obtain the given answer | **A1** | [3] |
| **(ii)** | Convert integrand into the form $A + \dfrac{B}{u+1}$ | **M1*** | |
| | Obtain integrand $A = 1$, $B = -2$ | **A1** | |
| | Integrate and obtain $u - 2\ln(u+1)$ | **A1**$\checkmark$ **+ A1**$\checkmark$ | |
| | Substitute limits correctly in an integral containing terms $au$ and $b\ln(u+1)$, where $ab \neq 0$ | **DM1** | |
| | Obtain the given answer following full and correct working<br>[The f.t. is on $A$ and $B$.] | **A1** | [6] |
| **7 (i)** | State modulus $2\sqrt{2}$, or equivalent | **B1** | |
| | State argument $-\frac{1}{3}\pi$ (or $-60°$) | **B1** | [2] |
| **(ii) (a)** | State answer $3\sqrt{2} + \sqrt{6}\,\mathrm{i}$ | **B1** | |
| **(b)** | *EITHER:* Substitute for $z$ and multiply numerator and denominator by conjugate of i$z$ | **M1** | |
| | Simplify the numerator to $4\sqrt{3} + 4\,\mathrm{i}$ or the denominator to 8 | **A1** | |
| | Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$ | **A1** | |
| | *OR*: Substitute for $z$, obtain two equations in $x$ and $y$ and solve for $x$ or for $y$ | **M1** | |
| | Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$ | **A1** | |
| | Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$ | **A1** | [4] |
| **(iii)** | Show points $A$ and $B$ in relatively correct positions | **B1** | |
| | Carry out a complete method for finding angle $AOB$, e.g. calculate the argument of $\dfrac{z^*}{\mathrm{i}z}$ | **M1** | |
| | Obtain the given answer | **A1** | [3] |
| **8 (i)** | State or imply the form $\dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$ | **B1** | |
| | Use a correct method to determine a constant | **M1** | |
| | Obtain one of $A = 2$, $B = 1$, $C = -1$ | **A1** | |
| | Obtain a second value | **A1** | |
| | Obtain a third value | **A1** | [5] |

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| **(ii)** | Use correct method to find the first two terms of the expansion of $(x+2)^{-1}$, $(1+\frac{1}{2}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$<br>Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction<br>Multiply out fully by $Bx + C$, where $BC \neq 0$<br>Obtain final answer $\frac{3}{4}-\frac{1}{4}x+\frac{5}{16}x^2$, or equivalent<br>[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A*, *B*, *C*.]<br>[In the case of an attempt to expand $(3x^2+x+6)(x+2)^{-1}(x^2+4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.] | **M1**<br><br>**A1**√ **+ A1**√<br>**M1**<br>**A1** | <br><br><br><br>[5] |
| **9 (i)** | Differentiate both equations and equate derivatives<br>Obtain equation $\cos a - a\sin a = -\frac{k}{a^2}$<br>State $a\cos a = \frac{k}{a}$ and eliminate $k$<br>Obtain the given answer showing sufficient working | **M1***<br>**A1 + A1**<br>**DM1**<br>**A1** | <br><br><br>[5] |
| **(ii)** | Show clearly correct use of the iterative formula at least once<br>Obtain answer 1.077<br>Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775) | **M1**<br>**A1**<br>**A1** | <br><br>[3] |
| **(iii)** | Use a correct method to determine $k$<br>Obtain answer $k = 0.55$ | **M1**<br>**A1** | <br>[2] |
| **10 (i)** | Express general point of $l$ in component form e.g. $(1+2\lambda,\ 2-\lambda,\ 1+\lambda)$<br>Using the correct process for the modulus form an equation in $\lambda$<br>Reduce the equation to a quadratic, e.g. $6\lambda^2+2\lambda-4=0$<br>Solve for $\lambda$ (usual requirements for solution of a quadratic)<br>Obtain final answers $-\mathbf{i}+3\mathbf{j}$ and $\frac{7}{3}\mathbf{i}+\frac{4}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$ | **B1**<br>**M1***<br>**A1**<br>**DM1**<br>**A1** | <br><br><br><br>[5] |
| **(ii)** | Using the correct process, find the scalar product of a direction vector for $l$ and a normal for $p$<br>Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\frac{2}{3}$<br>State a correct equation in any form, e.g. $\frac{2a-1+1}{\sqrt{(a^2+1+1)}.\sqrt{(2^2+(-1)^2+1)}}=\pm\frac{2}{3}$<br>Solve for $a^2$<br>Obtain answer $a = \pm 2$ | **M1**<br>**M1**<br>**A1**<br>**M1**<br>**A1** | <br><br><br><br>[5] |